Faculty of Health, Science and Technology

Syllabus

Post graduate level

Course Approval
The syllabus was approved by the Faculty of Health, Science and Technology on 2013-04-26 (Reg No: HNT 2013/22), and is valid from the spring semester of 2013 at Karlstad University.

Code
2FYS014

Doctoral study subject
Physics

Course name (english and swedish)
Lie Groups and Lie Algebras
Liegrupper och Liealgebror

Credit points
7,5 hp/ECTS credits

Language of instruction
Swedish or english

Degree level
Doctor

Target group and prerequisities
Admission to doctoral studies in physics or mathematics, or a Master's degree in physics or mathematics, or equivalent qualifications.

Aims
The course's goal is that the students acquire both basic and in-depth knowledge about Lie groups, Lie algebras and related mathematical structures and methods which arise e.g. in the description of symmetries in physics.

After completion of the course the student is expected to be able to

- identify continuous symmetries in mechanical, quantum mechanical and quantum field theoretical systems and describe them with the help of the relevant mathematical structures

- define the notions of Lie group, Lie algebra, simple Lie algebra, Kac-Moody algebra, representation, module, highest weight module, enveloping algebra etc

- give a detailed account of central aspects of the structure theory and representation theory of Lie algebras, such as nilpotency, solvability, semisimplicity, irreducible representations, Verma modules, dual representations, tensor product, root and weight diagrams, the Killing form, the Cartan-Weyl basis etc

- describe the construction of affine Lie algebras via loop algebras and central extension

- describe the classification of finite-dimensional representations of complex semisimple Lie algebras, as well as compute their characters with the Weyl character formula

- give an account of the relation between Lie groups and Lie algebras

- present the classification of finite-dimensional complex simple Lie algebras and of affine Lie algebras, as well as summarize the most important steps of the proof of the classification

- explain the conection between real and complex semisimple Lie algebras

- apply computer programs for performing calculations for complex simple Lie algebras and their representations.

Course content
The course is taught in the form of lectures, exercise sessions, self-study of advanced topics both individually and in small groups, as well as literature studies.

The following topics are treated:

- definition as well as structure theory and representation theory of finite-dimensional Lie groups and Lie algebras as well as of Kac-Moody algebras

- the classification of finite-dimensional complex simple Lie algebras and of affine Lie algebras, Cartan-Weyl bases, roots and simple roots, the Chevalley basis

- the Weyl group and root systems, root and weight diagrams, the enveloping algebra and Verma modules, irreducible highest weight modules, the Weyl and Weyl-Kac character formulas

- dual representations, tensor product of representations, the Racah-Speiser algorithm for decomposing tensor products

- Lie groups, relations between Lie groups and Lie algebras, Maurer-Cartan theory

- Real semisimple Lie algebras

Reeding list
See separate document.

Examination
The examination of the course is done in the form of homework exercises, written and oral presentations of advanced topics as well as an oral exam.

Course certificate
A course certificate is issued on request.

Quality Assurance
The purpose of the course management is to promote a continuous dialogue on teaching processes and on the fulfillment of learning outcomes. A written evaluation is performed at the end of the course, combined with a discussion between students and teachers of their experiences with all pertinent aspects of the course.
Course evaluations are compiled by the responsible department in accordance with the quality assurance procedures laid down by the Faculty and are made available to the Faculty Board, no later than one semester after completion of the course.

Grades
One of the grades Fail (U) or Pass (G) is awarded in the examination of the course.

Additional information

Reading list

Course name: Lie Groups and Lie Algebras, 2FYS014, 7,5 hp/ECTS (Doctoral studies)

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J. Fuchs and C. Schweigert: *Symmetries, Lie Algebras and Representations,*
Cambridge University Press, ISBN 9780521541190

Reference literature:

P. Cvitanovic: *Group Theory - Classics Illustrated*, http://www.cns.gatech.edu/GroupTheory